Mark Scheme (Results)

Summer 2007

GCE

GCE Mathematics

Mechanics M3 (6679)

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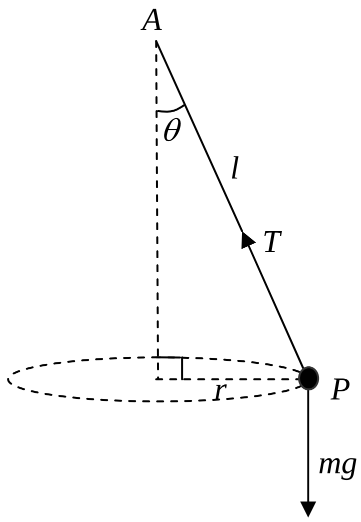
June 2007
6679 Mechanics M3
Mark Scheme

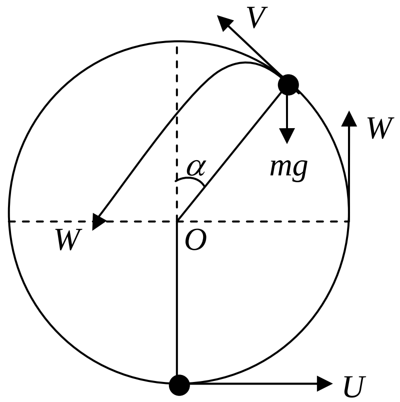
Question Number	Scheme	Marks
1.	(a) $A = \int_0^2 \left(2x - x^2\right) \mathrm{d}x$	M1 A1
	$= \left[x^2 - \frac{x^3}{3} \right]_{\dots}^{\dots}$	A1
	$A = \left[x^2 - \frac{x^3}{3} \right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$ * cso	A1 **(4)**
	(b) $\bar{x} = 1$ (by symmetry)	B1
	$\frac{4}{3}\bar{y} = \frac{1}{2}\int y^2 \,\mathrm{d}x = \frac{1}{2}\int \left(2x - x^2\right)^2 \mathrm{d}x$	M1
	$= \frac{1}{2}\int \left(4x^2 - 4x^3 + x^4\right) \mathrm{d}x$	A1
	$= \frac{1}{2}\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]$	A1
	$\frac{4}{3}\bar{y} = \frac{1}{2}\left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]_0^2 = \frac{8}{15}$	
	$\bar{y} = \frac{8}{15} \times \frac{3}{4} = \frac{2}{5}$ accept exact equivalents	A1 **(5)**
		[9]

Question Number	Scheme	Marks
2.	(a) Base, Cylinder, Container	
	Mass ratios: πh^2, $2\pi h^2$, $3\pi h^2$ Ratio of 1 : 2 : 3	B1
	$\bar{y}$: 0, $\frac{h}{2}$, $\bar{y}$	B1
	$3\pi h^2 \times \bar{y} = 2\pi h^2 \times \frac{h}{2}$	M1 A1
	Leading to $\bar{y} = \frac{1}{3}h$ * cso	A1 **(5)**
	(b) Liquid, Container, Total	
	Mass ratios: M, M, $2M$ Ratio of 1 : 1 : 2	B1
	$\bar{y}$: $\frac{h}{2}$, $\frac{h}{3}$, $\bar{y}$	B1
	$2M \times \bar{y} = M \times \frac{h}{2} + M \times \frac{h}{3}$	M1 A1
	$\bar{y} = \frac{5}{12}h$	A1 **(5)**
		[10]

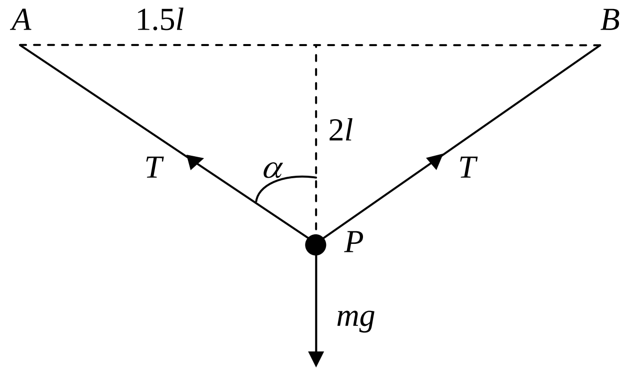
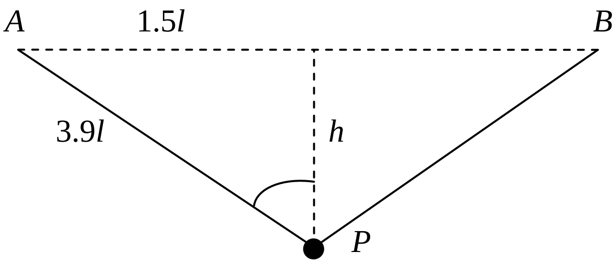
Question Number	Scheme	Marks
3.	(a) At surface	
	$\frac{k}{R^2} = mg \quad \Rightarrow \quad k = mgR^2 \quad *$ cso	M1 A1 **(2)**
	(b) N2L $\quad m\ddot{x} = -\frac{mgR^2}{x^2}$	
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{gR^2}{x^2}$ or $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right) = -\frac{gR^2}{x^2}$	M1
	$\int v\,\mathrm{d}v = -gR^2\int\frac{1}{x^2}\mathrm{d}x$ or $\frac{1}{2}v^2 = -gR^2\int\frac{1}{x^2}\mathrm{d}x$	M1
	$\frac{1}{2}v^2 = \frac{gR^2}{x} \; (+C)$	A1
	$x = 2R, v = 0 \quad \Rightarrow \quad C = -\frac{gR}{2}$	M1 A1
	$v^2 = \frac{2gR^2}{x} - gR$	
	At $x = R$, $\quad v^2 = \frac{2gR^2}{R} - gR$	M1
	$v = \sqrt{(gR)}$	A1 **(7)**
		[9]

Question Number	Scheme		Marks	
4.				
	$\uparrow \quad T\cos\theta = mg$		M1 A1	
	$\leftarrow \quad T\sin\theta = \dfrac{mv^2}{r}$		M1 A1	
	$\tan\theta = \dfrac{r}{\sqrt{(l^2 - r^2)}}$	or equivalent	M1 A1	
	$\tan\theta = \dfrac{v^2}{rg}$	Eliminating T	M1	
	$\dfrac{r}{\sqrt{(l^2 - r^2)}} = \dfrac{v^2}{rg}$	Eliminating θ	M1	
	$gr^2 = v^2\sqrt{(l^2 - r^2)}$ $\ast$	cso	A1	**(9)**
				[9]

Question Number	Scheme	Marks
5.	(a) $\ddot{x} = -\omega^2 x \quad \Rightarrow \quad 1 = \omega^2 \times 0.04 \quad (\Rightarrow \quad \omega = 5)$	M1 A1
	$T = \dfrac{2\pi}{5}$ awrt 1.3	A1 **(3)**
	(b) $v^2 = \omega^2\left(a^2 - x^2\right) \quad \Rightarrow \quad 0.2^2 = 5^2\left(a^2 - 0.04^2\right)$ ft their ω	M1 A1ft
	$a = \dfrac{\sqrt{2}}{25}$ accept exact equivalents or awrt 0.057	A1 **(3)**
	(c) Using $x = a\cos\omega t$	
	$\dfrac{1}{2}a = a\cos\omega t$ ft their ω	M1 A1ft
	$5t = \dfrac{\pi}{3}$	
	$t = \dfrac{\pi}{15}$	A1
	$T' = 4t = \dfrac{4\pi}{15}$ awrt 0.84	M1 A1 **(5)**
		[11]
	Alternative to (c)	
	Using $x = a\sin\omega t$	
	$\dfrac{1}{2}a = a\sin\omega t$ ft their ω	M1 A1ft
	$5t = \dfrac{\pi}{6}$	
	$t = \dfrac{\pi}{30}$	A1
	$T' = T - 4t = \dfrac{4\pi}{15}$ awrt 0.84	M1 A1 **(5)**

Question Number	Scheme		Marks	
6.				
	(a) Energy $\frac{1}{2}m\left(U^2 - v^2\right) = mga\left(1 + \cos\alpha\right)$		M1 A1=A1	
	□ $\left(T +\right)\ mg\cos\alpha = \frac{mv^2}{a}$		M1 A1	
	Leaves circle when $T = 0$			
	$g\cos\alpha = \frac{U^2 - 2ga - 2ga\cos\alpha}{a}$ Eliminating v		M1	
	Leading to $U^2 = ag\left(2 + 3\cos\alpha\right)$ ∗	cso	A1	**(7)**
	(b) Using conservation of energy from the lowest point of the surface			
	$\frac{1}{2}m\left(U^2 - W^2\right) = mga$		M1 A1=A1	
	$W^2 = U^2 - 2ag$			
	Using $\cos\alpha = \frac{1}{\sqrt{3}}$, $W^2 = ag\left(2 + \frac{3}{\sqrt{3}}\right) - 2ag$		M1	
	$= ag\sqrt{3}$ ∗	cso	A1	**(5)** **[12]**
	Alternatives for (b) *are given on the next page.*			

Question Number	Scheme	Marks
6.	*Alternative to part* (b) *using conservation of energy from the point where P loses contact with surface.*	
	$\left(V^2 = ag\cos\alpha = \frac{ga}{\sqrt{3}}\right)$	
	Energy $\quad \frac{1}{2}m\left(W^2 - V^2\right) = mga\cos\alpha$	M1 A1
	$\frac{1}{2}m\left(W^2 - \frac{1}{\sqrt{3}}ag\right) = mga \times \frac{1}{\sqrt{3}}$	A1
	Leading to $\quad W^2 = ag\sqrt{3}$ * $\quad$ cso	M1 A1 **(5)**
	Alternative to part (b) *using projectile motion from the point where P loses contact with surface.*	
	$V^2 = ag\cos\alpha = \frac{ga}{\sqrt{3}}$	
	$\downarrow \quad W_y^2 = V^2\sin^2\alpha + 2ga\cos\alpha$	
	$= \frac{1}{\sqrt{3}}ag\left(1 - \frac{1}{3}\right) + 2ga \times \frac{1}{\sqrt{3}} = \frac{8\sqrt{3}}{9}ag$	M1 A1
	$\leftarrow \quad V_x = V\cos\alpha$	A1
	$W^2 = W_y^2 + V_x^2 = \frac{8\sqrt{3}}{9}ag + \frac{1}{3}ag\sqrt{3} \times \frac{1}{3} = ag\sqrt{3}$ * $\quad$ cso	M1 A1 **(5)**

Question Number	Scheme	Marks
7.	(a) 	
	$AP = \sqrt{\left((1.5l)^2 + (2l)^2\right)} = 2.5l$	M1 A1
	$\cos\alpha = \dfrac{4}{5}$	B1
	Hooke's Law $T = \dfrac{\lambda(2.5l - 1.5l)}{1.5l} \left(= \dfrac{2\lambda}{3}\right)$	M1 A1
	$\uparrow$ $2T\cos\alpha = mg$ $\left(T = \dfrac{5mg}{8}\right)$	M1 A1
	$2 \times \dfrac{2\lambda}{3} \times \dfrac{4}{5} = mg$ $\left(\dfrac{2\lambda}{3} = \dfrac{5mg}{8}\right)$	M1
	$\lambda = \dfrac{15mg}{16}$ $\ast$ cso	A1 **(9)**
	(b) 	
	$h = \sqrt{\left((3.9l)^2 - (1.5l)^2\right)} = 3.6l$	M1 A1
	Energy $\dfrac{1}{2}mv^2 + mg \times h = 2 \times \dfrac{15mg}{16} \times \dfrac{(2.4l)^2}{2 \times 1.5l}$ ft their h	M1 A1ft = A1
	Leading to $v = 0$ $\ast$ cso	A1 **(6)**
		[15]